MAT 2379 - Spring 2011
Assignment 6 : Solutions

7.1 (2 points) $SE_{\bar{y}_1} = \frac{s_1}{\sqrt{n_1}} = \frac{4.3}{\sqrt{6}} = 1.7555; SE_{\bar{y}_2} = \frac{s_2}{\sqrt{n_2}} = \frac{5.7}{\sqrt{12}} = 1.6454$
$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE_{\bar{y}_1}^2 + SE_{\bar{y}_2}^2} = \sqrt{1.7555^2 + 1.6454^2} = 2.4061$

7.8 (2 points) $SE_{\bar{y}_1} = \frac{s_1}{\sqrt{n_1}} = \frac{0.400}{\sqrt{9}} = 0.13333; SE_{\bar{y}_2} = \frac{s_2}{\sqrt{n_2}} = \frac{0.220}{\sqrt{6}} = 8.9815 \times 10^{-2}$

$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE_{\bar{y}_1}^2 + SE_{\bar{y}_2}^2} = \sqrt{0.13333^2 + (8.9815 \times 10^{-2})^2} = 0.16076$

7.30 (6 points) a) H_0 : mean tibia length is independent of gender $\mu_1 = \mu_2$
H_A :mean tibia length depends on gender $\mu_1 \neq \mu_2$

$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE_{\bar{y}_1}^2 + SE_{\bar{y}_2}^2} = \sqrt{\frac{2.87^2}{60} + \frac{3.52^2}{50}} = 0.62056$; Formula 7.1 gives $df = 94.3. We$ also have $t_s = \frac{78.42-80.44}{0.62056} = -3.2551$ From Table 4, $t_{0.005} = 2.626$, $t_{0.0005} = 3.390$. Hence $0.001 < P - value < 0.01$ and we reject H_0

b) There is sufficient evidence to conclude that mean tibia length is larger in females than in males

c) Given the overlap of the distributions from the means and standard deviations, we could not be confident about the sex.

d) Formula 7.1 gives $df = 7.8$. $t_s = -1.03$ and $0.20 < P - value < 0.40$ and we do not reject H_0

7.54 (3 points) Let 1 denote drug and 2 denote placebo

(a) $H_0 : \mu_1 = \mu_2$ vs $H_A : \mu_1 > \mu_2$

Compute $SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE_{\bar{y}_1}^2 + SE_{\bar{y}_2}^2} = \sqrt{\frac{12.05^2}{25} + \frac{13.78^2}{25}} = 3.6611$

$t_s = \frac{31.96-25.32}{3.6611} = 1.8137$. Formula 7.1 gives $df = 47.2$. From Table 4, $t_{0.04} = 1.787$, $t_{0.03} = 1.924$. Hence $0.03 < P - value < 0.04$ and we reject H_0 and conclude the drug is effective.

b) The only change is that we would double the $p-value$ and then based on the evidence, the drug would not be effective at the 5% significance level

7.60 (3 points) A 95% confidence interval for the difference $\mu_1 - \mu_2$ is given by

$$\bar{y}_1 - \bar{y}_2 \pm t_{0.025} SE_{(\bar{y}_1-\bar{y}_2)} \tag{1}$$

Here formula 4.1 gives $df = 5.7$ and $SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE_{\bar{y}_1}^2 + SE_{\bar{y}_2}^2} = \sqrt{\frac{21^2}{4} + \frac{27^2}{4}} = 17.103$, From Table 4,$t(6)_{0.025} = 2.447$ and $\bar{y}_1 - \bar{y}_2 = 106 - 102 = 4.0$. The confidence interval becomes

$$(-37.9, 45.9) \tag{2}$$

7.85(3 points) a) $SE_{\bar{y}_1} = \frac{s_1}{\sqrt{n_1}} = \frac{9.6}{\sqrt{12}} = 2.7713, SE_{\bar{y}_2} = \frac{s_2}{\sqrt{n_2}} = \frac{10.2}{\sqrt{13}} = 2.8290$

$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE^2_{\bar{y}_1} + SE^2_{\bar{y}_2}} = \sqrt{2.7713^2 + 2.8290^2} = 3.9602$

b)$SE_{\bar{y}_1} = \frac{s_1}{\sqrt{n_1}} = \frac{2.7}{\sqrt{22}} = 0.57564, SE_{\bar{y}_2} = \frac{s_2}{\sqrt{n_2}} = \frac{1.9}{\sqrt{19}} = 0.43589$

$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE^2_{\bar{y}_1} + SE^2_{\bar{y}_2}} = \sqrt{0.57564^2 + 0.43589^2} = 0.72205$

c) $SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE^2_{\bar{y}_1} + SE^2_{\bar{y}_2}} = \sqrt{1.2^2 + 1.4^2} = 1.8439$

9.4(3 points) a) Let 1 denote treated side and 2 denote control side. The standard error is

$SE_{(\bar{y}_1-\bar{y}_2)} = \frac{s_d}{\sqrt{n_d}} = \frac{1.118}{\sqrt{15}} = 0.28867$; $t\,(14)_{0.025} = 2.145$.

The 95% confidence interval is

$$\bar{d} \pm 2.145 \frac{s_d}{\sqrt{n_d}} \tag{3}$$

That is

$$0.117 \pm 2.145\,(0.28867) \tag{4}$$
$$(-0.50, 0.74) \tag{5}$$

b) Using the wrong approach we would get

$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{SE^2_{\bar{y}_1} + SE^2_{\bar{y}_2}} = \sqrt{\frac{1.217^2}{15} + \frac{1.302^2}{15}} = 0.46017$

The 95% confidence interval would be using $df = 28$

$$\bar{d} \pm 2.048\,(0.46017) \tag{6}$$

That is

$$(-0.83, 1.06) \tag{7}$$

This interval is wider.

9.19 (3 points) Let p denote the probability that a patient will have fewer minor seizures with valproate than with a placebo. Set $H_0 : p = 0.5$ vs $H_A : p > 0.5$.

$N_+ = 14$, $N_- = 5$, $B_s = 14$. From Table 7, with $n_d = 19$ we see that

$$0.025 < p - value < 0.05 \tag{8}$$

and we reject H_0

9.31 (3 points)

H_0 : Weight change is not affected by treatment $\mu_1 = \mu_2$

H_A :Treatment does affect weight change $\mu_1 \neq \mu_2$

We compute differences and take their absolute values. The ranks of the absolute differences are

$$6, 7, 8, 1, 3, 9, 5, 4, 2 \tag{9}$$

The signed ranks are

$$6, -7, -8, -1, -3, -9, 5, 4, -2 \tag{10}$$

Thus $W_+ = 6 + 5 + 4 = 15$, $W_- = 7 + 8 + 1 + 3 + 9 + 2 = 30$

$W_s = \max(15, 30) = 30$. From Table 8 with $\mathrm{n}_d = 9$, we find $p - value >$ 0.20 and H_0 is not rejected

Total= 28 points